Estimating Latent Variable Graphical Models with Moments and Likelihoods

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Percy Liang

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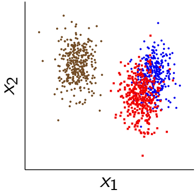
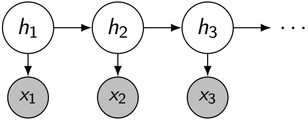
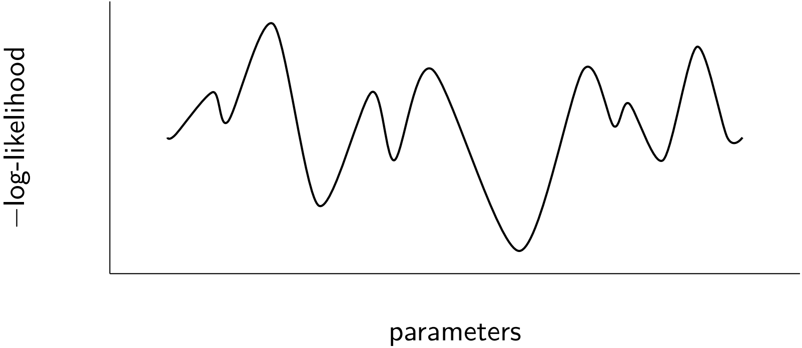
Latent Variable Graphical Models

- Gaussian Mixture Models
- Latent Dirichlet Allocation
- Hidden Markov Models
- PCFGs
- ...





Parameter Estimation is Hard



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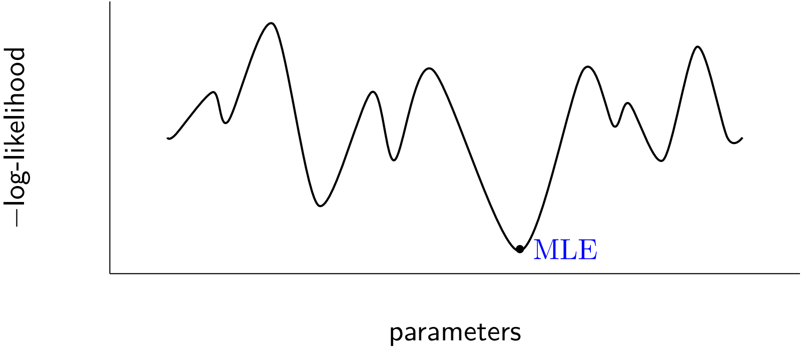

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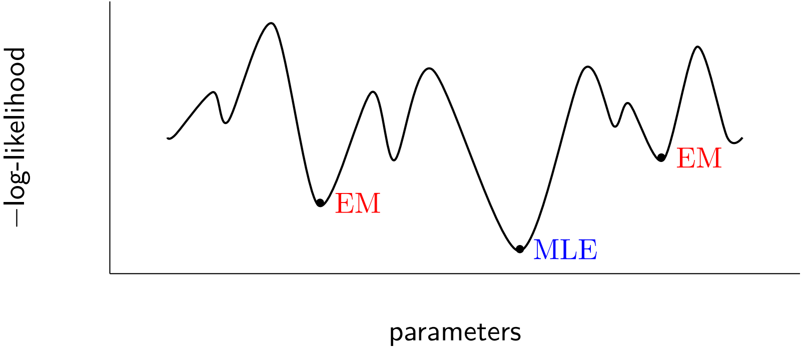

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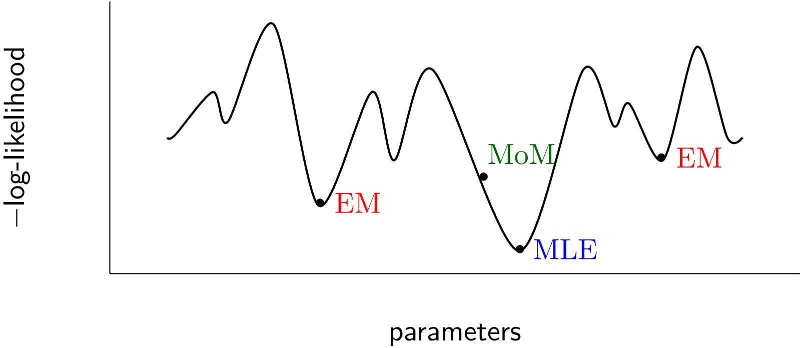

- Log-likelihood function is non-convex.
- MLE is consistent but intractable.
- Local methods (EM, gradient descent, ...) are tractable but inconsistent.
- *Method of moments* estimators can be consistent and computationally-efficient, but require more data.

Consistent estimation for general models

- Several estimators based on the method of moments.
  - **Phylogenetic trees:** Mossel and Roch 2005.
  - **Hidden Markov models:** Hsu, Kakade, and Zhang 2009
  - **Latent Dirichlet Allocation:** Anandkumar et al. 2012
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- **How can we apply the method of moments to estimate *parameters efficiently* for a *general* model?**

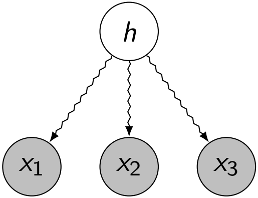
Setup

- Discrete models, d, k.
- Assume $d > k$.
- Parameters and marginals can be put into a matrix or tensor -¿ introduce notation.
- Assume infinite data.
- Highlight directed vs undirected - we focus on directed.

Background: Three-view Mixture Models

Definition (Bottleneck)

A hidden variable h is a **bottleneck** if there exist three observed variables (**views**) x_1, x_2, x_3 that are *conditionally independent* given h.



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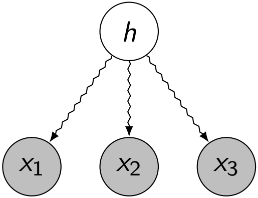
- Anandkumar, Hsu, and Kakade 2012 provide an algorithm to estimate conditional moments $\mathbb{P}(x_i \mid h)$ based on tensor eigendecomposition.
- In general, three views are necessary for identifiability (Kruskal 1977).

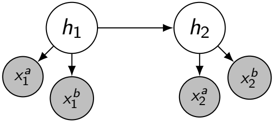


Outline

TODO: Make outline a diagram

Example: a bridge, take I

- Each edge has a set of parameters.



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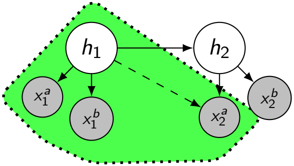

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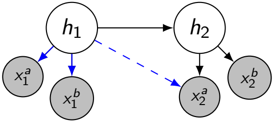

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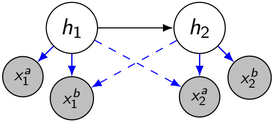

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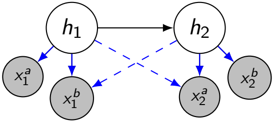
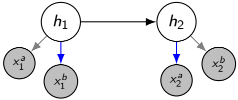
- Each edge has a set of parameters.
- h_1 and h_2 are bottlenecks.
- We can learn $\mathbb{P}(x_1^a|h_1), \mathbb{P}(x_1^b|h_1), \cdots$.
- However, we can't learn $\mathbb{P}(h_2|h_1)$ this way.



Example: a bridge, take II

- Observe the joint distribution, TODO: Use cartoon matrices

$$\underbrace{\mathbb{P}(x_1^b, x_2^a)}_{M_{12}} = \sum_{h_1, h_2} \underbrace{\mathbb{P}(x_1^b \mid h_1)}_{O^{(1|1)}} \underbrace{\mathbb{P}(x_2^a \mid h_2)}_{O^{(2|2)}} \underbrace{\mathbb{P}(h_1, h_2)}_{Z_{12}}.$$



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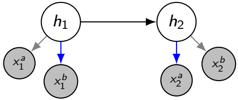

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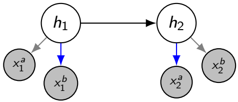

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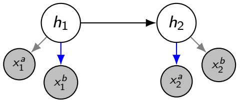
- $\mathbb{P}(h_2 \mid h_1)$ can be recovered by normalization.



Outline

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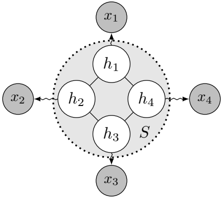
Exclusive Views

Definition (Exclusive views)

We say $h_i \in S \subseteq \mathbf{h}$ has an **exclusive view** x_v if

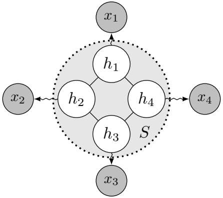
1. There exists some observed variable x_v which is conditionally independent of the others $S \backslash \{h_i\}$ given h_i.
2. The conditional moment matrix $O^{(v|i)} \triangleq \mathbb{P}(x_v \mid h_i)$ has full column rank k and can be recovered.
3. TODO: Exclusive views for a clique



Exclusive views give parameters

- Given *exclusive views*, $\mathbb{P}(x \mid h)$, learning cliques is solving a linear equation! TODO: Use cartoon tensors

$$\underbrace{\mathbb{P}(x_1, \ldots, x_m)}_{M} = \sum_{h_1,\ldots,h_m} \underbrace{P(x_1|h_1)}_{O^{(1|1)}} \cdots \underbrace{P(h_1, \cdots, h_m)}_{Z}$$



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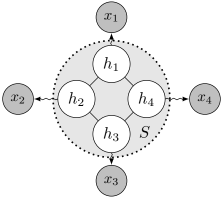

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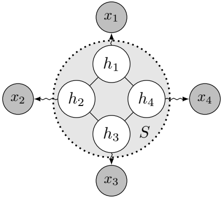
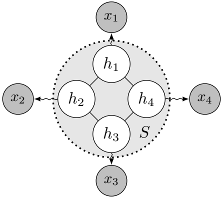


Bottlenecked graphs

- When are we assured exclusive views?



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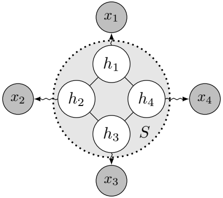

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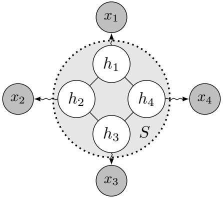
- **Theorem:** A bottlenecked clique has exclusive views.

  TODO: Say show in paper.



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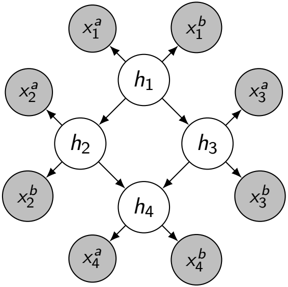

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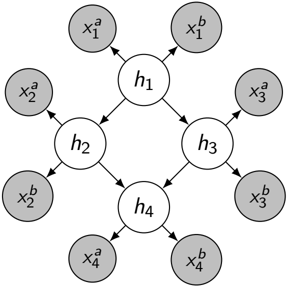

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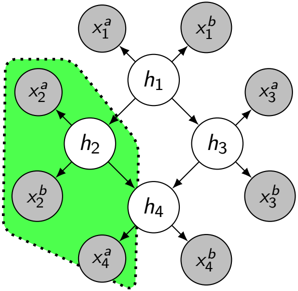

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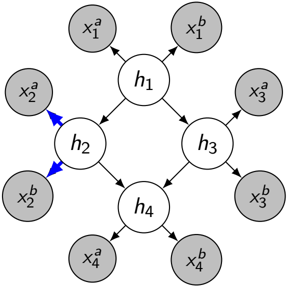

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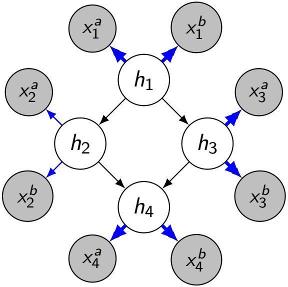

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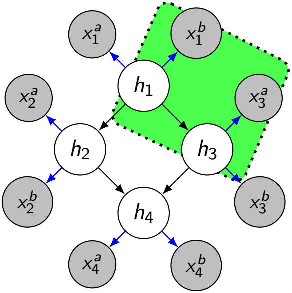

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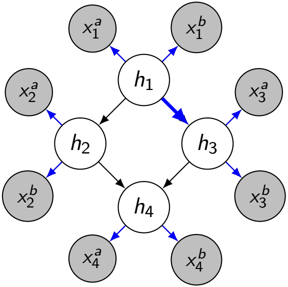

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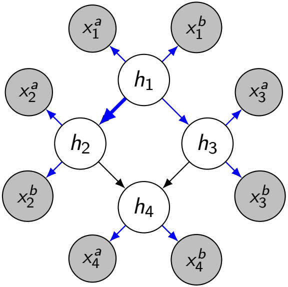

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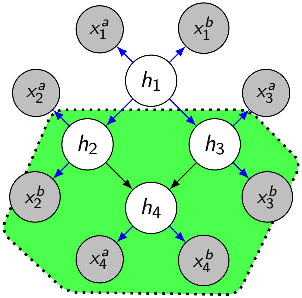

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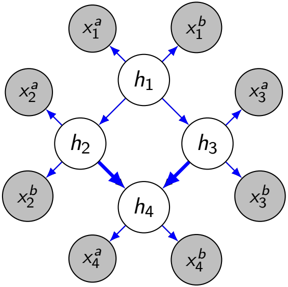

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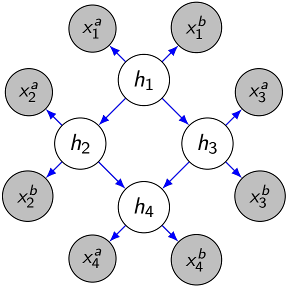
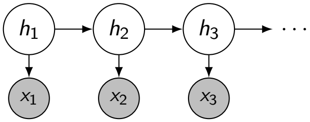
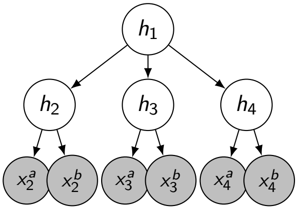


More Bottlenecked Examples

Hidden Markov models



Latent Tree models



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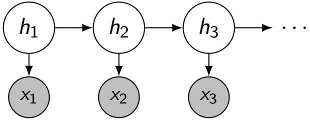

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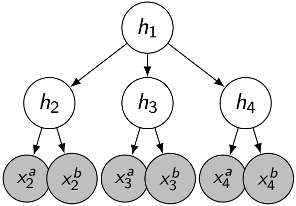
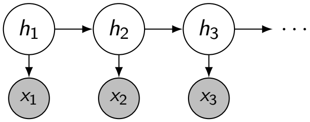


More Bottlenecked Examples

Halpern and Sontag 2013

Hidden Markov models



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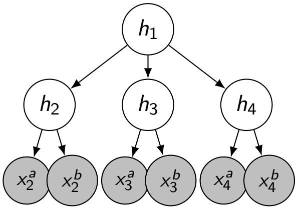
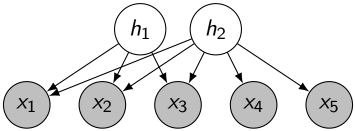


Noisy Or (non-example)



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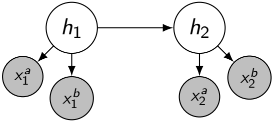
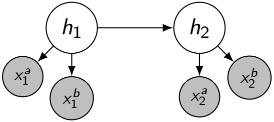
Convex marginal likelihoods

- The MLE is statistically most efficient, but usually non-convex.



$$\log \mathbb{P}(\mathbf{x}) = \log \sum_{h_1,h_2} \mathbb{P}(\mathbf{x}_1|h_1)\,\mathbb{P}(\mathbf{x}_2|h_2)\mathbb{P}(h_1, h_2)$$

Convex marginal likelihoods



- The MLE is statistically most efficient, but usually non-convex.
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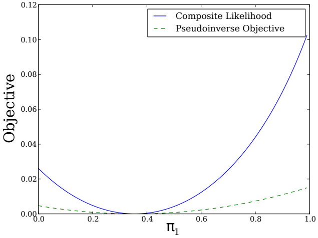

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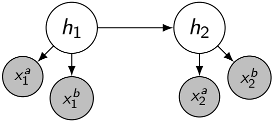

- The MLE is statistically most efficient, but usually non-convex.
- If we fix the conditional moments, $-\log \mathbb{P}(x)$ is convex in θ.
- No closed form solution, but a local method like EM is guaranteed to converge to the global optimum.

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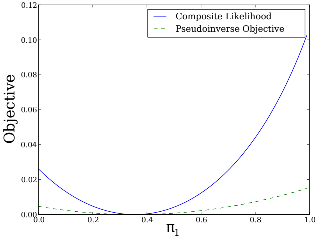

Composite likelihoods

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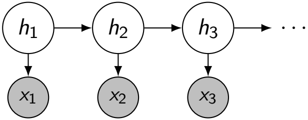

$$\log \mathbb{P}(\mathbf{x}) = \log \sum_{h_1, h_2, h_3} \underbrace{\mathbb{P}(\mathbf{x}_1 \mid h_1)\, \mathbb{P}(\mathbf{x}_2 \mid h_2) \mathbb{P}(\mathbf{x}_3 \mid h_3)}_{\text{known}}$$

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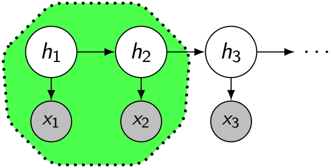

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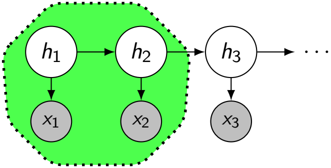

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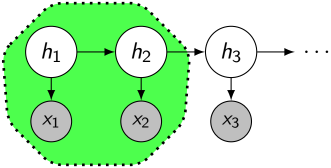
$$\color{blue}{\mathbb{P}(h_1, h_2)}$$

Composite likelihoods

- In general, the full likelihood is still non-convex. TODO: Specify which x.
- Consider *composite likelihood* on a subset of observed variables.
- Can be shown that estimation with composite likelihoods is consistent (Lindsay 1988).
- Asymptotically, the composite likelihood estimator is more efficient.



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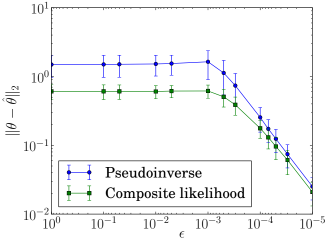


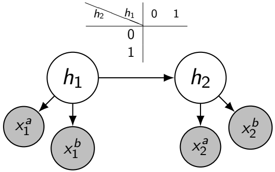
Outline

TODO: Make outline a diagram

Recovering parameters in directed models

- Conditional probability tables are the default parameterization for a directed model.
- Can be recovered by normalization:

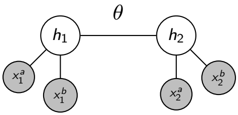
$$\mathbb{P}(h_2 \mid h_1) = \frac{\mathbb{P}(h_1, h_2)}{\sum_{h_2} \mathbb{P}(h_1, h_2)}.$$



Recovering parameters in undirected log-linear models

- Assume a log-linear parameterization,
  TODO: use sum over cliques - talk through.

$$p_\theta(\mathbf{x}, \mathbf{h}) = \exp\left(\theta^\top \phi(\mathbf{x}, \mathbf{h}) - A(\theta)\right).$$



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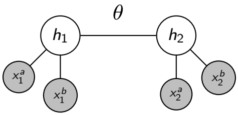
$$\mathcal{L}_{\text{unsup}}(\theta) \triangleq \mathbb{E}_{\mathbf{x}\sim\mathcal{D}}[-\log \sum_{\mathbf{h}\in\mathcal{H}} p_\theta(\mathbf{x}, \mathbf{h})].$$



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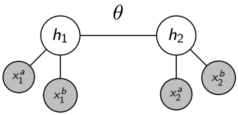


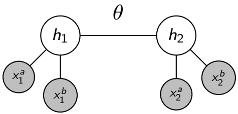
- However, the *supervised* negative log-likelihood is convex,

$$\mathcal{L}_{\text{sup}}(\theta) \triangleq \mathbb{E}_{(\mathbf{x},\mathbf{h})\sim\mathcal{D}_{\text{sup}}}[-\log p_\theta(\mathbf{x}, \mathbf{h})]$$
$$= -\theta^\top \left(\sum_{\mathcal{C}\in\mathcal{G}} \mathbb{E}_{(\mathbf{x},\mathbf{h})\sim\mathcal{D}_{\text{sup}}}[\phi(\mathbf{x}_\mathcal{C}, \mathbf{h}_\mathcal{C})] \right) + A(\theta).$$

Recovering parameters in undirected log-linear models

- Recall, the marginals can typically estimated from supervised data.

$$\mathcal{L}_{\text{sup}}(\theta) = -\theta^\top \underbrace{\left(\sum_{\mathcal{C} \in \mathcal{G}} \mathbb{E}_{(\mathbf{x},\mathbf{h}) \sim \mathcal{D}_{\text{sup}}}[\phi(\mathbf{x}_\mathcal{C}, \mathbf{h}_\mathcal{C})] \right)}_{\mu_\mathcal{C}} + A(\theta).$$



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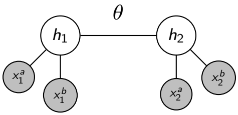


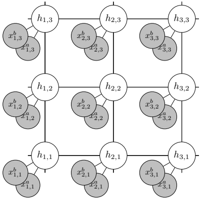
- However, the marginals can also be *consistently* estimated by moments!

$$\mu_\mathcal{C} = \sum_{\mathbf{x}_\mathcal{C}, \mathbf{h}_\mathcal{C}} \underbrace{\mathbb{P}(\mathbf{x}_\mathcal{C} \mid \mathbf{h}_\mathcal{C})}_{\text{cond. moments}} \underbrace{\mathbb{P}(\mathbf{h}_\mathcal{C})}_{\text{hidden marginals}} \phi(\mathbf{x}_\mathcal{C}, \mathbf{h}_\mathcal{C}).$$

Optimizing pseudolikelihood

- Estimating marginals $\mu_{\mathcal{C}}$ is independent of treewidth, but computing the normalization constant is: TODO: convex but not easy

$$A(\theta) \triangleq \log \sum_{\mathbf{x},\mathbf{h}} \exp\left(\theta^\top \phi(\mathbf{x}, \mathbf{h})\right).$$



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- We can use pseudolikelihood (**besag75pseudo**) to consistently estimate distributions over local neighborhoods.

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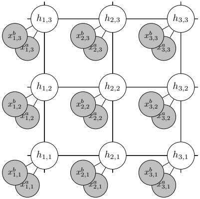

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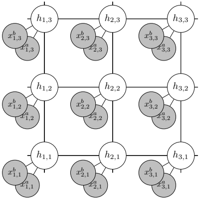
- Clique marginals not sufficient statistics, but we can still estimate them.



Conclusions

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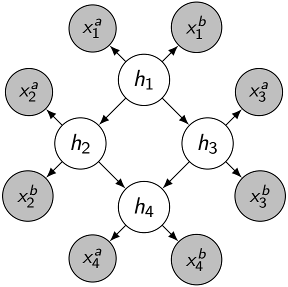
- TODO: Show the venn diagram on progress on generality..
- An algorithm for any (non-degenerate) **bottlenecked discrete graphical models**.



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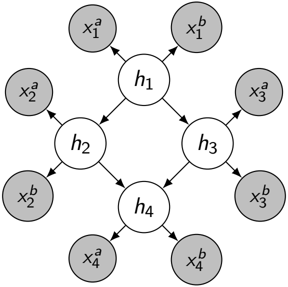

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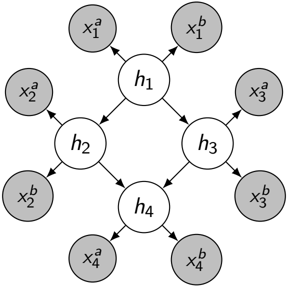

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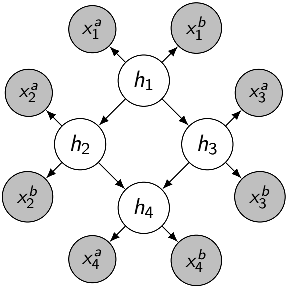
- Combine moment estimators with composite likelihood estimators.
- Extends to **log-linear models**.
  - Allows for easy regularization, missing data,



Thank you!